References

[1] C. H. Durney, C. C. Johnson, P. W. Barber, H. Massoudi, M. F. Iskander, J. L. Lords, D. K. Ryser, S. J. Allen, and J. C. Mitchell, *Radiofrequency Radiation Dosimetry Handbook* 2nd ed., Departments of Electrical Engineering and Bioengineering, University of Utah, Salt Lake City, UT, 1978.

[2] C. C. Johnson, C. H. Durney, and H. Massoudi, "Long-wavelength electromagnetic power absorption in prolate spheroidal models of man and animals," *IEEE Trans. Microwave Theory Tech.*, vol. MTT-23, pp. 739–747, Sept. 1975.

[3] C. H. Durney, C. C. Johnson, and H. Massoudi, "Long-wavelength analysis of plane wave irradiation of a prolate spheroid model of man," *IEEE Trans. Microwave Theory Tech.*, vol. MTT-23, pp. 246–254, Feb. 1975.

[4] H. Massoudi, C. H. Durney, and C. C. Johnson, "Long-wavelength electromagnetic power absorption in ellipsoidal models of man and animals," *IEEE Trans. Microwave Theory Tech.*, vol. MTT-25, pp. 47–52, Jan. 1977.

[5] P. W. Barber, "Resonance electromagnetic absorption by nonspherical dielectric objects," *IEEE Trans. Microwave Theory Tech.*, vol. MTT-25, pp. 373–381, May 1977.

[6] O. P. Gandhi, "Conditions of strongest electromagnetic power deposition in man and animals," *IEEE Trans. Microwave Theory Tech.*, vol. MTT-23, pp. 1021–1029, Dec. 1975.

[7] A. W. Guy, J. F. Lehmann, and J. B. Stonebridge, "Therapeutic applications of electromagnetic power," *Proc. IEEE*, vol. 62, pp. 55–75, 1974.

[8] D. L. Conover, W. H. Parr, E. L. Sensintaffar, and W. E. Murray, Jr., *Measurement of Electric and Magnetic Field Strengths from Industrial Radiofrequency (15-40.68 MHz) Power Sources*, C. C. Johnson and M. L. Shore, Eds., HEW Publication (FDA) 77-8011, Dec. 1976.

[9] P. F. Wacker and R. R. Bowman, "Quantifying hazardous electromagnetic fields: Scientific basis and practical considerations," *IEEE Trans. Microwave Theory Tech.*, vol. MTT-19, pp. 178–187, Feb. 1971.

[10] P. C. Waterman, "Symmetry, unitary, and geometry in electromagnetic scattering," *Phys. Rev. D.*, vol. 3, pp. 825–839, 1971.

[11] P. W. Barber and C. Yeh, "Scattering of electromagnetic waves by arbitrary shaped dielectric bodies," *Appl. Opt.*, vol. 14, 1975, pp. 2864–2872.

[12] J. D. Jackson, *Classical Electrodynamics*. New York: Wiley, 1962.

[13] A. Hizal and Y. K. Baykal, "Heat potential distribution in an inhomogeneous spherical model of a cranial structure exposed to microwaves due to loop or dipole antennas," *IEEE Trans. Microwave Theory Tech.*, vol. MTT-26, pp. 607–612, Aug. 1978.

[14] H. Chew, P. J. McNutly, and M. Kerker, "Model for raman and fluorescent scattering by molecules embedded in small particles," *Phys. Rev. A.*, vol. 13, 1976, pp. 396–404.

[15] P. M. Morse and H. Feshbach, *Methods of Theoretical Physics*. New York: McGraw-Hill, 1953.

[16] M. F. Iskander, H. Massoudi, C. H. Durney, and S. J. Allen, "Measurements of the RF power absorption in human and animal phantoms exposed to near-field radiation," to be published.

Short Paper

A General Equivalent Network of the Input Impedance of Symmetric Three-Port Circulators

G. BITTAR AND GY. VESZELY

Abstract—**Starting from the network model of ferrite-filled resonators, a general equivalent network of the input impedance of symmetric, three-port circulators is given. The main advantage of the network, that it contains the original elements of the resonator model, so the physics of operation can be clearly seen and the results of field analysis can be directly used.**

I. The Network Model of Ferrite-Filled Resonators

Hammer [1] gave the network model of ferrite-filled resonators. If the resonator has a three-fold symmetry axis, the excitations are on magnetic wall and only two resonator modes are taken into account, then the network model can be seen in Fig. 1. The two ports marked by φ are nonreciprocal phase shifters. They have the characteristic as follows:

$$I_1 = -e^{-j\varphi}I_2,\ U_1 = e^{-j\varphi}U_2,\ \varphi = 0, \qquad \pm 2\pi/3. \qquad (1)$$

The values of Z_r^+, Z_r^- can be obtained from the eigenvalues

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The authors are with the Department of Theoretical Electricity, Technical University, 1521 Budapest, Hungary.

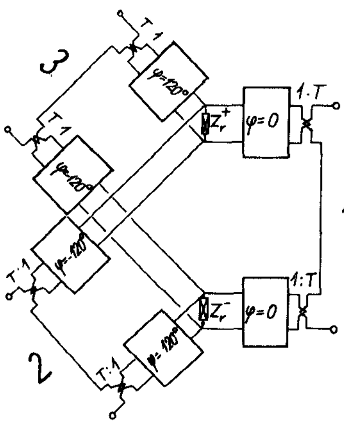

Fig. 1. The resonator model.

and losses of the resonator, and the transformer ratio T is obtainable from the eigenfunctions of the resonator and those of the coupling transmission lines [1]. The impedance matrix of the three port in Fig. 1 is

$$Z = \begin{bmatrix} Z_1 & Z_2 & Z_3 \\ Z_3 & Z_1 & Z_2 \\ Z_2 & Z_3 & Z_1 \end{bmatrix} \qquad (2)$$

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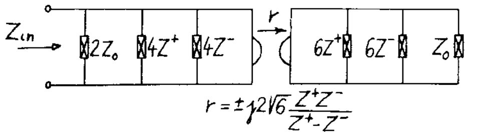

Fig. 2. General equivalent network of input impedance.

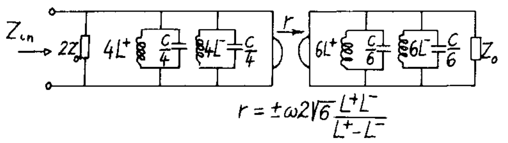

Fig. 3. Equivalent network of the input impedance of stripline circulator.

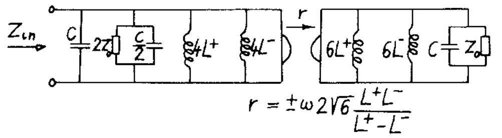

Fig. 4. Equivalent network of the input impedance of lumped-element circulator.

where

$$Z_1 = Z^+ + Z^-$$
$$Z_2 = Z^+ e^{-j2\pi/3} + Z^- e^{j2\pi/3}$$
$$Z_3 = Z^+ e^{j2\pi/3} + Z^- e^{-j2\pi/3} \qquad (3)$$

where

$$Z^+ = T^2 Z_r^+, \; Z^- = T^2 Z_r^-. \qquad (4)$$

II. The Input Impedance of the Terminated Three-Port and its Equivalent Network

Let the ports 2 and 3 of the three-port (2) be terminated by a complex impedance Z_0. The input impedance of port 1 can be expressed in the following form:

$$Z_{in} = -Z_0 + \frac{(Z_1+Z_0)^3 + Z_2^3 + Z_3^3 - 3(Z_1+Z_0)Z_2Z_3}{(Z_1+Z_0)^2 - Z_2Z_3}. \qquad (5)$$

Substituting (3) into (5) after a simple but lengthy algebra we obtain

$$Z_{in} = Z_0 \frac{6Z^+Z^- + Z^+Z_0 + Z^-Z_0}{3Z^+Z^- + 2Z^+Z_0 + 2Z^-Z_0 + Z_0^2}. \qquad (6)$$

It can be easily seen that the input impedance of the network in Fig. 2 is the same as (6).

The network in Fig. 2 is the main result of this paper. It contains the elements Z^+, Z^- of the resonator model, so it is of general validity. Z^+, Z^- can be complex quantities, so the losses can be included in the model. If external tuning elements are used Z_0 is complex too.

The general conditions of the perfect circulation can be easily obtained from Fig. 2.

1) Let $Z^+ = jX$ and $Z^- = -jX$ (i.e., the parallel equivalent of $6Z^+$ and $6Z^-$, $4Z^+$ and $4Z^-$ is infinite);

2) If 1) is fulfilled, then the matching condition $Z_{in} = Z_0$ is fulfilled, if $r^2 = 2Z_0^2$, i.e., $4.6(X^2/2X)^2 = 2Z_0^2$, i.e., $X/Z_0 = 1/\sqrt{3}$, which is the well-known result.

III. Examples

In the case of a lossless stripline circulator Z_0 is real, while Z^+ and Z^- are imaginary and Fig. 2 is transformed to the network in Fig. 3. The values of L^+, L^-, and C in accordance to [1] are

$$L^{\pm} = \frac{\mu_{eff} R\psi}{\pi} \frac{2}{x_{11}^2 - 1} \frac{1}{\left(1 \mp \frac{\kappa}{\mu}\frac{1}{x_{11}^2-1}\right)^2}$$

$$C = \frac{\varepsilon\pi R}{\psi x_{11}^2} \frac{x_{11}^2 - 1}{2} \qquad (7)$$

where R is the disk radius, ψ is the coupling angle, $x_{11} = 1.84$, and μ, κ are Polder tensor elements. Using these values the two conditions of the perfect circulation are in agreement with the well-known results [2], [3].

The lumped-element circulator can be regarded as a ferrite filled resonator excited below the lowest resonance frequency. The approximate network model of such a resonator contains only two inductances L^+, L^-. These elements are not sufficient for tuning the circulator, therefore complex terminal admittances $1/Z_0 + j\omega C$ are used at ports 2 and 3, and a parallel capacity C at port 1. The network model of the input impedance can be seen in Fig. 4. The conditions of the perfect circulation are the same as in [4], taking into account, that $L^{\pm} = \mu^{\pm}\xi/3$, where ξ is the geometrical factor defined in [4].

The table below shows a comparison between the bandwidth computed by [4] and that one computed from Fig. 4. (The latter is exact in the frame of the given model.)

κ/μ	0.5	0.4	0.3	0.2	0.1
$\Delta\omega/\omega$ [4]	0.16	0.13	0.10	0.068	0.035
$\Delta\omega/\omega$ Fig. 4	0.15	0.12	0.089	0.059	0.029

References

[1] G. Hammer, "The analysis of the waveguide *Y* circulator," *Periodica Polytechnica, Elec. Eng.*, vol. 14, pp. 61–76, 1970.

[2] H. Bosma, "On stripline *Y* circulation at UHF," *IEEE Trans Microwave Theory Tech.*, vol. MTT-12, pp. 61–72, 1964.

[3] C. E. Fay and R. L. Comstock, "Operation of the ferrite junction circulator," *IEEE Trans. Microwave Theory Tech.*, vol. MTT-13, pp. 15–27 Jan. 1965.

[4] Y. Konishi, "Lumped element *Y* circulator," *IEEE Trans. Microwave Theory Tech.*, vol. MTT-13, pp. 852–864, Nov. 1965.